Introduction to Kinematics

Kinematics is the branch of mechanics that describes the motion of objects using words, diagrams, numbers, graphs, and equations. It focuses on the following key concepts:

1. Scalars and Vectors:

o Scalars are quantities that have only magnitude (e.g., distance, speed).

o Vectors are quantities that have both magnitude and direction (e.g., displacement, velocity, acceleration).

2. Distance and Displacement:

o Distance is a scalar quantity that refers to “how much ground an object has covered” during its motion.

o Displacement is a vector quantity that refers to “how far out of place an object is”; it is the object’s overall change in position.

3. Speed and Velocity:

o Speed is a scalar quantity that refers to “how fast an object is moving.”

S = distance/time

o Velocity is a vector quantity that refers to “the rate at which an object changes its position.”

Velocity = displacement /time

4. Acceleration:

o Acceleration is a vector quantity that refers to “the rate at which an object changes its velocity.”

Acceleration = velocity/time

Equations of Motion

In kinematics, several key equations describe the motion of objects under uniform acceleration:

1. v = u + at

( v ): final velocity ( u ): initial velocity ( a ): acceleration ( t ): time

1. s = ut + 1/2at^2 `

( s ): displacement

1. v^2 = u^2 + 2as

These equations help in solving problems related to the motion of objects.

Graphical Representation

Kinematics also involves the use of graphs to represent motion:

Slope of graph is the tangent to the curve in the graph at that point

Slope of graph = `tantheta = text(quantity on y axis)/text(quantity on x axis)`

• Position-Time Graphs: Show how position changes over time.

Slope gives speed(distance/time)

• Velocity-Time Graphs: Show how velocity changes over time.

Slope gives accleration

• Acceleration-Time Graphs: Show how acceleration changes over time.

Understanding these graphs is crucial for visualizing and analyzing motion.

Instantaneous Velocity

Instantaneous velocity is the velocity of an object at a specific moment in time. It is a vector quantity, meaning it has both magnitude and direction. Mathematically, it is defined as the derivative of the position function with respect to time.

Formula:

[ `v\_{t} = (dx\_{t})/(dt)` ] where:

• ( v(t) ) is the instantaneous velocity at time ( t )

• ( x(t) ) is the position function with respect to time ( t )

Instantaneous Acceleration

Instantaneous acceleration is the acceleration of an object at a specific moment in time. It is also a vector quantity and is defined as the derivative of the velocity function with respect to time.

Formula:

` A\_{t} = (dv\_{t})/(dt)` where:

• ( a(t) ) is the instantaneous acceleration at time ( t )

• ( v(t) ) is the velocity function with respect to time ( t )

Alternatively, since velocity is the derivative of position, instantaneous acceleration can also be expressed as the second derivative of the position function: [ `a\_{t} = (d^2x)/((dt)^2) `]

Understanding the Concepts

• Instantaneous Velocity: Imagine you’re driving a car and you look at the speedometer at a particular instant. The speedometer reading gives you the instantaneous velocity of the car at that moment.

• Instantaneous Acceleration: If you suddenly press the accelerator or brake, the change in the speedometer reading over a very short time interval gives you the instantaneous acceleration.

a = (dv)/(dt) = (dv)/(dt) \* (dx)/(dx) = (dx)/(dt) \* (dv)/(dx) = v(dv)/(dx)

integration

if acceleration is given as a function of x [a = f(x)]

for example `a = sin x`

find velocity from x = 0 to x =1

`a = v(dv)/(dx) = sinx`

`vdv = sinxdx`

Integrate on both sides

`int \_0^v vdv = int\_0^1sinxdx`

`[v^2/2]\_{0}^v = [-cosx]\_{0}^1`

`[v^2/2 – 0/2] = [(-cos1) – (-cos0)]`

`v^2/2 = -cos1 + 1`

`v^2/2 = 1 – cos1`

`v = sqrt(2(1 – cos1))`

Free Fall

Free fall refers to the motion of an object under the influence of gravitational force only, with no other forces acting on it (such as air resistance). This type of motion is characterized by a constant acceleration due to gravity, denoted as ( g ), which is approximately ( 9.81 , \text{m/s}^2 ) near the Earth’s surface.

Keep in mind

• Initial Velocity: In free fall, the initial velocity (( u )) is often zero if the object is dropped from rest.

• Acceleration: The only acceleration acting on the object is due to gravity (( g )).

• Time of Fall: The time it takes for an object to fall can be calculated using the displacement formula if the height is known.

Relative motion

Imagine two trains moving in oppsitye direction one in north direction and other in south direction you are sitting in the trsain going to south moving at a speed of 40 km per hr aaaaaand other train moving 30 km per hr. sitting in the window seat you will feel the train is moving faster than 30 km/hr this is due to relative motion

Here for you the train is moving at 70km/hr how? Beacause you are moving(frame of reference is moving) so to calculate the speed of the train to north

N

S

40km/hr

30km/hr

We consider a 30 km/hr to the oppsite side of the north train in order for us to be at rest

(because according to us the we are not moving the things around us are moving)

So we feel the train to south is moving at 40 + 30 = 70km/hr

But for a an observer standing at the station and whachting these two trains he feels them moving only at 40 and 30 because he is not moving(frame of reference(his eyes) is at rest)

So

Two objects opposite direction

`V\_{rel} = v\_{1} – (-v\_{2}) = v\_{1} + v\_{2}`

Two object moving in same direction

`V\_{rel} = v\_{1} – v\_{2}`

River flow

Here the swimmer starts from Q aiming for P but reaches R due to flow of river called drift of position

V\_{c} is the velocity of the river

V\_{x} is the horizontal component of the swimmer here given by v\_{c} and v\_{y} is the vertical component. here V is the velecoty of swimmer in still water

P

drift

R

Vc

Vc

V

Q

Here `v\_{x} = V\_{c}`

Net speed = `sqrt(V\_{x}^2 + V\_{y}^2)`

Time to cross river `t = d/V\_{y}`

Vc

Vc

Q

Drift(d)

R

P

θ

V

Simmer jumps making an angle `theta` with the normal

`V\_{w.r.t to earth} = V + V\_{c}`

(from trianglelaw of vector addition)

Here velocity along river

`V\_{x} = V\_{c} – Vsintheta`

In perpendicular dirction

`V\_{y} = Vcostheta`

The resultant velocity to cross river

`V\_{R} = sqrt(V\_{x}^2 + V\_{y}^2)`

Time taken to cross = `t = d/Vcostheta`

The drift by the river

`X = V\_{x}\*t = (V\_{c} – Vsintheta )\*d/(Vcostheta)`

If question asked to reduce theta or drift speed use `(dx)/(dtheta) = 0`

Will give the minimum of `theta` value

If the swimmer jumps downstream (along flow of river)

Q

R

P

Vc

V

Vc

θ

Only change in V\_{x}

`V\_{x} = V\_{c} + Vsintheta`

`X = V\_{x}\*t = (V\_{c} + Vsintheta )\*d/(Vcostheta)`

Remaining all properties same

Rain

Motion in two dimension

velocity

`V\_{x} = (dx)/(dt) = vcostheta`

`V\_{y} = (dy)/(dt) = vsintheta`

The instantaneous velocity of particle

`Vec v = v\_{x} hati + v\_{y} hatj`

Net velocity

`V = sqrt(v\_{x}^2 +v\_{y}^2)`

The angle formed by the trajectory with positive x- direction is

`tantheta = v\_{y}/v\_{x}`

`theta = tan^(-1)v\_{y}/v\_{x}`

Acceleration

`a\_{x} = (dv\_{x})/(dt) = v\_{x}(dv\_{x})/(dx) = d^2x/(dt)^2`

`a\_{y} = (dv\_{y})/(dt) = v\_{y}(dv\_{y})/(dy) = d^2y/(dt)^2`

Instantaneous acceleration

`vec a\_{text(net)} = a\_{x}hat i + a\_{y} hat j`

Magnitude

`a\_{text(net)} = sqrt(a\_{x}^2 + a\_{y}^2)`

Direction of net acceleration

`phi = tan^(-1) a\_{y}/a\_{x}`

(ɸ and θ are different, θ is direction for velocity)

Trajectory of a particle in two dimensional motion

`y = f(x)`

(just have to make an equation relating x and y coordinates iof the particle

Projectile motion

To analyze the motion we resolve the motion of the body in two separate one dimensional motions. One along x-direction and other along y-direction. We resolve the initial velocity in two corresponding directions

The horizontal component of the initial velocity is `u\_{x} = u \* cos theta`

The vertical component is

`u\_{y} = u \* sin theta`

In the whole motion there is only one force acting on the body i.e. the force of gravity due to which it has only one acceleration in y-direction "-g".

If we consider the horizontal projection of the body during flight, it will run with a constant velocity from the starting point O to the point where the projectile will hit the ground. In y-direction motion particle starts with the velocity u sine and retarded by g. It goes up to a maximum height H and then it returns to the ground. If these two motions are combined, it results the trajectory shown in figure-1.35.

If the body is projected at time t = 0 it will fall on the ground at time `t = T\_{f}` known as time of flight, the value of which can be given as

Using

`s = ut - 1/2 \* g \* t ^ 2`

We take  `0 = u \* sin theta\*T\_{f} - 1/2 \* g \* T\_{f} ^ 2 T\_{f} = (2u \* sin theta)/g`

When the particle is at the topmost point of its trajectory its vertical component of velocity is zero and as Hbe the maximum height, at which it will have only the horizontal component of velocity

We get v ^ 2 = u ^ 2 - 2gs 0 = (u \* sin theta) ^ 2 - 2gH H = (u ^ 2 \* sin^2 theta)/(2g) ...(1.39)

The horizontal distance to which the body travels during its flight is known as the horizontal range, which can be evaluated to be the distance traveled by the horizontal projection of the body in the duration time of flight.

The horizontal range of projectile is R = u \* cos theta \* T\_{f} R = (u ^ 2 \* sin 2theta)/g ...(1.40) From equation-(1.40), it is clear that the horizontal range of projectile depends on the angle of projection theta as e varies, range will change and range will accordingly be maximum when the factor sin20 will be have a maximum value. Thus the maximum range is

When

R = (u ^ 2)/g

sin 2theta = 1

If range is not maximum, then R depends on sin20, and there can be two values of theta at which sin 2theta has a single value for (0 < theta < 90 deg) This implies when range is not maximum and there will always be two values of angles of projections, at which we get the same ranges, if u is same in both the cases. These two angles are known as complementary angles as to be theta\_{1} + theta\_{2} = 90 deg

When a body is projected at t = 0 then at time t = t the velocity projections of the particle in x and y directions are v coso and v sino, respectively as shown in figure-1.37, if velocity at time r is v and it is making an angle o with the positive direction of x-axis.

In x-direction velocity component is

v\_{x} = mu \* cos theta

(Remains constant as a\_{x} = 0 )

In y-direction velocity component is v\_{y} = u \* sin theta - gt

(Retarded by a\_{y} = - g )

In vectorial form velocity of the particle in projectile motion as a function of time is given as overline v =(u cos theta) hat i + (u \* sin theta - gt) hat j

Its magnitude at time / is

v = sqrt(u ^ 2 + g ^ 2 \* t ^ 2 - 2ugt \* sin theta) (1.42)

During motion we can also find the projectile coordinate at a general time t = tas

Its x-coordinate is x= u \* cos theta .t (As a\_{x} = 0 ) ... (1.43)

As particle moves in x-direction with constant velocity u \* cos theta

Its y-coordinate is

y = u \* sin theta \* t - 1/2 \* g \* t ^ 2 (1.44)

As in y-direction, particle's initial velocity is u sine and is retarded by g.

Eliminate / between equations-(1.43) and (1.44), we get the relation in x and y.

y = x \* tan theta - (g \* x ^ 2)/(2u ^ 2 \* cos^2 theta) (1.45)

It is the equation of the path of trajectory in the coordinate system where x-direction is along horizontal and y-direction is along vertical. This trajectory path equation is very useful in solving numerical problems. Let us take few examples on basic projectile motion